Geometric constructions and elements of Galois' theory
List 10. Splitting fields of polynomials

1. Check that the splitting field of a quadratic polynomial $ax^2+bx+c \in Q[x]$ is the field $Q(\sqrt{\Delta})$, where $\Delta = b^2 - 4ac$.
2. Verify that the field $Q(\sqrt{-3})$ is the splitting field of the polynomial $x^3 - 1$.
3. Although any two essentially distinct irreducible polynomials from $Q[x]$ always have disjoint sets of roots, it may happen that they have the same splitting fields. Find an example of such two irreducible polynomials of degree 2, for both of which the field $Q(\sqrt{3})$ is their splitting field.
4. Verify that for any $a, b, c \in Q$ the splitting fields of the polynomials $ax^2 + bx + c$ and $cx^2 + bx + a$ are the same. And what about the polynomials $a_n x^n + \ldots + a_0$ and $a_0 x^n + \ldots + a_n$ of arbitrary degree n?
5. Justify that the field $Q(\sqrt[4]{5})$ is **not** the splitting field of the polynomial $x^4 - 5$. Find and list all roots of this polynomial, and describe explicitely its splitting field.