Foundation of geometry and non-euclidean geometry
List 2+. "Arithmetic" models of geometry.

1. Consider the following model of the incidence geometry. Points are interpreted as all pairs (x, y) of real numbers, and lines are interpreted as equations of the following two kinds:
   (1) all equations of form $x = c$ for any real parameter $c \in R$;
   (2) all equations of form $y = (x - a)^2 + b$ for any parameters $a, b \in R$.
   The relation of incidence is interpreted as the fact that a pair of numbers satisfies an equation. Verify whether the incidence axioms I1, I2 and I3, and the parallel axiom R, hold in this model.
2. Consider a modified Hilbert's arithmetic model in which we replace real numbers with rational ones. It means that points in this model are pairs (x, y) of rational numbers, and lines are sets of points that satisfy an equation of form $ax + by + c = 0$ with rational coefficients a, b, c. The remaining primitive notions are interpreted in the same way as in the ordinary Hilbert's model. Check which of the axioms of euclidean geometry are (and which are not) fulfilled in this modified model.
3. Consider a model of the theory of incindence geometry in which points are all pairs (x, y) of real numbers, and lines are the sets of pairs of the following three kinds:
   (a) $\{(x, y) : y = c\}$, for any real constant c;
   (b) $\{(x, y) : x = c\}$, for any realm constant c;
   (c) $\{(x, y) : y = ax + b \text{ and } x \geq 0\} \cup \{(x, y) : y = 2ax + b \text{ and } x \leq 0\}$, for any real constants $a \neq 0$ and $b \in R$.
   Show that in this model the incidence axioms I1-I3 and the parallel axiom R are satisfied.
4. Show that the model from the previous exercise is (as a model of incidence theory) **isomorphic** to the Hilbert's model. To do this, construct (and write with an explicit formula) a bijective map $F : R^2 \to R^2$ which sends the lines of the Hilbert's model to the lines of the model from Exercise 3, giving a one-to-one correspondence between these sets of lines.
5. Consider the model of the incidence theory in which points are all pairs (x, y) of real numbers, and lines are the sets of pairs satisfying equations of the following two kinds:
   (a) $x = c$ for any real parameter c;
   (b) $y = a|x - b| + c$ for any parameter $a > 0$ and any real parameters b and c.
   Check which of the axioms of the incidence theory are satisfied in this model. Does the Lobachevsky Axiom hold in this model?